Aptitude
Ratio, Proportion and Partnership

Concept 1: Ratio.

As we already know, that ratio is the comparison of two Quantities by division or the relation that one quantity bears to another with respect to magnitude.

If a and b are two numbers, then the ratio a to b is a/b or a÷b and is denoted by a:b. The two quantities that are being compared are called terms.

The first is called antecedent and the second term is called consequent.

As always, we would be focusing on the underlying concepts, rather than memorizing formulas, following are the types of ratios you would encounter.

i.) **Duplicate Ratio:** The ratio of squares of two numbers is called the duplicate ratio of the two numbers.
For example: $\frac{2^2}{5^2} = \frac{4}{25}$ is called the duplicate ratio of $\frac{2}{5}$.
In the same way we can understand the Triplicate ratio in which the cubes of the two numbers are called the triplicate ratio of two numbers.

ii.) **Sub – Duplicate ratio:** The ratio of the square root of two numbers is called the sub- duplicate ratio of the numbers.
$\frac{2}{3}$ is sub-duplicate ratio of $\frac{4}{9}$.

Now, you guys give me an example of sub-triplicate ratio in the same way, as discussed here.

iii.) **Inverse ratio or Reciprocal ratio:** If the antecedent and consequent of a ratio are interchanged, the new ratio is called the inverse ratio of the first.
$\frac{2}{5}$ *is the inverse ratio of* $\frac{5}{2}$ **.**

iv.) **Compound Ratio:** When we compound two or more ratios with each other through product or multiplication, the result is simply a compound ratio. Thus, the product of two or more ratios; i.e, ab:cd is a ratio compounded of the simple ratios a:c and b:d. Required compounded ratio = (2/3 x 6/11 x 11/2) = 2/1.

These terms are just for informative purposes, so that you do not get confused on not being familiar with the terminology.

Now, Let's see some examples and test our learning till now.

Ex.1.) The ratio of boys to girls in a class is 2:3. If total students are 120.Find the number of boys in the class.

Ans.) Here, we see that the ratio of boys to girls is 2:3. Therefore we can deduce that for every 2 boys in the class, we have 3 girls. So, if we consider 2 boys and 3 girls as a group, we have 5 students in total in that group.

Now, as per the question, since we need to calculate the number of boys present, we try to frame a new ratio of boys to the total students in the class.
For that, we get $\frac{Boys}{(Boys+girls)} = \frac{2}{2+3} = \frac{2}{5}$.

That means, in 5 students we have 2 boys, hence in 120 students, we will have $\frac{2}{5}$ x 120 = 48 boys.

Ex.2.) The present ratio of ages of A and B is 4:5. 18 years ago, this ratio was 11:16. Find the sum of their present ages.

Ans.) Let the present ages are 4x:5x , then it is given that

$\frac{4x-18}{5x-18} = \frac{11}{16}$

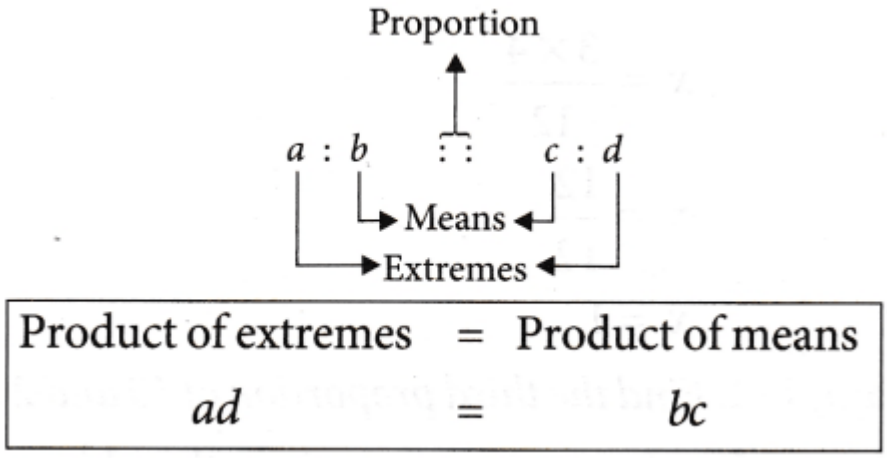
- ⇨ 64x-288=55x-198
- ⇨ 9x=90
- ⇨ X=10.

Therefore, ages 40 and 50.
Sum = 90.

Concept 2: Proportion:

As already discussed, the equality of two ratios is called proportion if $\frac{a}{b} = \frac{c}{d}$ then a,b,c and d are said to be in proportion and we write a:b::c:d.

Now, let's discuss the terms related to proportion, and eventually we will see some examples at the end.
But, an important concept, that needs to be discussed is as follows :
If four quantities are in proportion, then Product of means = Product of extremes, for example, in proportion a:b::c:d, we have b x c= a x d.



i.) **Mean proportional:** If a : x : : x : b , x is called the mean or second proportional of a & b.

We have $\frac{a}{x} = \frac{x}{b}$ or $x^2=ab$ or $x=\sqrt{ab}$

Therefore, mean proportional of a and b is $\sqrt{ab}$.

ii.) **Third proportional:** If a:b :: b:x , x is called the third proportional to a and b.

We have $\frac{a}{b} = \frac{b}{x}$ or $x = \frac{b^2}{a}$.

Suppose a:b :: c:d is a proportion of two ratios. We can write it as a:b = c:d. The quantity c is known as third proportional to the quantities a, b, and d.

iii.) **Fourth proportional:** If a:b :: c:x , x is called the fourth proportional of a, b and c.

$\frac{a}{b} = \frac{c}{x}$ or $x = \frac{b\,x\,c}{a}$, thus fourth proportional of a, b and c is $\frac{b\,x\,c}{a}$.

If $\frac{a}{b} = \frac{c}{d}$, then

1. Componendo:

$$\frac{a+b}{b} = \frac{c+d}{d}$$

2. Dividendo:

$$\frac{a-b}{b} = \frac{c-d}{d}$$

For componendo, we are just adding 1 to both sides of the equation, and for dividendo, 1 is being subtracted from both sides of the equation.

On combining the above two equations, we get the Componendo and Dividendo formula as :

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Ex.1.) The sum of two numbers is 15, and their ratio is 2/3. Find the numbers.

Ans.) Let the numbers be x and y,
So, x+y=15, Now, x/y=2/3.
Applying Componendo, $\frac{x+y}{y} = \frac{2+3}{3}$

- ⇨ 15/y= 5/3
- ⇨ Y = 9, x = 6 .

Concept 3 - Partnership:

We have discussed already that partnership requires partners, whether it is cricket or partners in business.
To run a business or any organisation, we need to have partnerships. There are two types of partners.
i.) **Working partners:** Those partners who invest the money and manage the business or affairs of an organisation are called working partners.
ii.) **Sleeping partners:** Those partners who merely invest money and do not involve themselves in business affairs are called sleeping partners.

In partnerships, we have mainly three types of conditions related to distribution of profit in business.

Condition 1:
When time period of investment is constant, then profit of partners will be divided into ratio C1 : C2 : C3 : ……Cn,
Where, C1, C2, C3, …..Cn are investments of respective partners.

Condition 2 :
When amount invested is same for different partners but time period is different, then profit will be divided in the ratio of their time invested, that is
t1 : t2 : t3…… : tn .

Condition 3:
When the amount of investment and time period both are different for different partners, then profit will be divided into the following ratio.
C1t1 : C2t2 : C3t3 : …… : Cntn.

In order to have a clear understanding of the above mentioned cases, lets take an example,

Ex.1.) Three partners undergo a partnership. Their investments are Rs.30000, Rs.45000 and Rs.50000 respectively for the time periods of 4 years, 2 years and 1 year respectively. Find the ratio of their profits.

Ans.) Here we see that the investments and time period of investments, both are different for all the partners, so we would are provide with Condition 3.

Using the same concept, the ratio of their profits would be,

C1t1 : C2t2 : C3t3

⇨ 30000 x 4 : 45000 x 2 : 50000 x 1

⇨ 30 x 4 : 45 x 2 : 50 x 1

⇨ 12 : 9 : 5

Ex.2.) A and B started a business with initial investments in the ratio 5:7 . If after one year their profits were in the ratio 1:2 and the period of A's investment was 7 months, B invested the money for ?

Ans.) Let investments of A and B respectively be 5x and 7x and period of B's investment be "t" months.

Then , $\frac{(5x)\ x\ 7}{(7x)\ x\ t} = \frac{1}{2}$

Therefore, we get, t = 10.

PROBLEMS ON RATIO, PROPORTION AND PARTNERSHIP
FOUNDATION

1.	A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?	
	A.	Rs. 460
	B.	Rs. 484
	C.	Rs. 550
	D.	Rs. 664

2.	Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:	
	A.	2 : 5
	B.	3 : 5
	C.	4 : 5
	D.	6 : 7

3.	A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?	
	A.	Rs. 500
	B.	Rs. 1500
	C.	Rs. 2000
	D.	None of these

4.	Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?	
	A.	2 : 3 : 4

	B.	6 : 7 : 8
	C.	6 : 8 : 9
	D.	None of these

5.	In a mixture 60 litres, the ratio of milk and water 2 : 1. If this ratio is to be 1 : 2, then the quantity of water to be further added is:	
	A.	20 litres
	B.	30 litres
	C.	40 litres
	D.	60 litres

MODERATE

1.	The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?	
	A.	8 : 9
	B.	17 : 18
	C.	21 : 22
	D.	Cannot be determined

2.	Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40 : 57. What is Sumit's salary?	
	A.	Rs. 17,000
	B.	Rs. 20,000
	C.	Rs. 25,500
	D.	Rs. 38,000

3.	If 0.75 : *x* :: 5 : 8, then *x* is equal to:

A.	1.12
B.	1.2
C.	1.25
D.	1.30

4. The sum of three numbers is 98. If the ratio of the first to second is 2 :3 and that of the second to the third is 5 : 8, then the second number is:

A.	20
B.	30
C.	48
D.	58

5. If Rs. 782 be divided into three parts, proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$, then the first part is:

A.	Rs. 182
B.	Rs. 190
C.	Rs. 196
D.	Rs. 204

HOTS-HIGH ORDER THINKING SKILLS

1. A and B invest in a business in the ratio 3 : 2. If 5% of the total profit goes to charity and A's share is Rs. 855, the total profit is:

A.	Rs. 1425
B.	Rs. 1500
C.	Rs. 1537.50

	D.	Rs. 1576

2.	A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest Rs. 6500 for 6 months, B, Rs. 8400 for 5 months and C, Rs. 10,000 for 3 months. A wants to be the working member for which, he was to receive 5% of the profits. The profit earned was Rs. 7400. Calculate the share of B in the profit.	
	A.	Rs. 1900
	B.	Rs. 2660
	C.	Rs. 2800
	D.	Rs. 2840

3.	A, B and C enter into a partnership in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his share 50%. If the total profit at the end of one year be Rs. 21,600, then B's share in the profit is:	
	A.	Rs. 2100
	B.	Rs. 2400
	C.	Rs. 3600
	D.	Rs. 4000

4.	A, B, C subscribe Rs. 50,000 for a business. A subscribes Rs. 4000 more than B and B Rs. 5000 more than C. Out of a total profit of Rs. 35,000, A receives:	
	A.	Rs. 8400
	B.	Rs. 11,900
	C.	Rs. 13,600
	D.	Rs. 14,700

5.	Three partners shared the profit in a business in the ratio 5 : 7 : 8. They had partnered for 14 months, 8 months and 7 months respectively. What was the ratio of their investments?

	A.	5 : 7 : 8
	B.	20 : 49 : 64
	C.	38 : 28 : 21
	D.	None of these

SOLUTIONS
FOUNDATION

1. Answer: Option **B**
Explanation:

$\frac{4}{15}A = \frac{2}{5}B$

$\Rightarrow A = \left(\frac{2}{5} \times \frac{15}{4}\right)B$

$\Rightarrow A = \frac{3}{2}B$

$\Rightarrow \frac{A}{B} = \frac{3}{2}$

$\Rightarrow$ A : B = 3 : 2.

$\therefore$ B's share = Rs. $\left(1210 \times \frac{2}{5}\right)$ = Rs. 484.

Answer: Option **B**

Explanation:

$\frac{4}{15}A = \frac{2}{5}B$

$\Rightarrow A = \left(\frac{2}{5} \times \frac{15}{4}\right)B$

$\Rightarrow A = \frac{3}{2}B$

$\Rightarrow \frac{A}{B} = \frac{3}{2}$

$\Rightarrow$ A : B = 3 : 2.

$\therefore$ B's share = Rs. $\left(1210 \times \frac{2}{5}\right)$ = Rs. 484.

2. Answer: Option **C**

Explanation:

Let the third number be x.

Then, first number = 120% of x =	$\frac{120x}{100}$	=	$\frac{6x}{5}$
Second number = 150% of x =	$\frac{150x}{100}$	=	$\frac{3x}{2}$

Ratio of first two numbers =	$\frac{6x}{5}$	:	$\frac{3x}{2}$	= 12x : 15x = 4 : 5.

3. Answer: Option **C**

Explanation:

Let the shares of A, B, C and D be Rs. 5x, Rs. 2x, Rs. 4x and Rs. 3x respectively.

Then, 4x - 3x = 1000

$\Rightarrow x$ = 1000.

$\therefore$ B's share = Rs. 2x = Rs. (2 x 1000) = Rs. 2000.

4. Answer: Option **A**

Explanation:

Originally, let the number of seats for Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.

Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$).

$$\Rightarrow \left(\frac{140}{100} \times 5x\right), \left(\frac{150}{100} \times 7x\right) \text{ and } \left(\frac{175}{100} \times 8x\right)$$

$$\Rightarrow 7x, \frac{21x}{2} \text{ and } 14x.$$

$$\therefore \text{ The required ratio} = 7x : \frac{21x}{2} : 14x$$

$\Rightarrow 14x : 21x : 28x$

$\Rightarrow 2 : 3 : 4.$

5. Answer: Option **D**

Explanation:

$$\text{Quantity of milk} = \left(60 \times \frac{2}{3}\right) \text{litres} = 40 \text{ litres}.$$

Quantity of water in it = (60- 40) litres = 20 litres.

New ratio = 1 : 2

Let quantity of water to be added further be x litres.

$$\text{Then, milk : water} = \frac{40}{20 + x}.$$

$$\text{Now, } \frac{40}{20 + x} = \frac{1}{2}$$

$\Rightarrow 20 + x = 80$

$\Rightarrow x = 60.$

$\therefore$ Quantity of water to be added = 60 litres.

MODERATE

1. Answer: Option **C**
Explanation:
Originally, let the number of boys and girls in the college be 7*x* and 8*x* respectively.
Their increased number is (120% of 7*x*) and (110% of 8*x*).

$$\Rightarrow \left(\frac{120}{100} \times 7x\right) \text{ and } \left(\frac{110}{100} \times 8x\right)$$

$$\Rightarrow \frac{42x}{5} \text{ and } \frac{44x}{5}$$

$$\therefore \text{ The required ratio} = \left(\frac{42x}{5} : \frac{44x}{5}\right) = 21 : 22.$$

2. Answer: Option **D**
Explanation:
Let the original salaries of Ravi and Sumit be Rs. 2*x* and Rs. 3*x* respectively.

$$\text{Then, } \frac{2x + 4000}{3x + 4000} = \frac{40}{57}$$

$\Rightarrow 57(2x + 4000) = 40(3x + 4000)$
$\Rightarrow 6x = 68{,}000$
$\Rightarrow 3x = 34{,}000$
Sumit's present salary = $(3x + 4000)$ = Rs.$(34000 + 4000)$ = Rs. 38,000.

3. Answer: Option **B**
Explanation:

$$(x \times 5) = (0.75 \times 8) \Rightarrow x = \left(\frac{6}{5}\right) = 1.20$$

4. Answer: Option **B**
Explanation:

Let the three parts be A, B, C. Then,

$$A : B = 2 : 3 \text{ and } B : C = 5 : 8 = \left(5 \times \frac{3}{3}\right) : \left(8 \times \frac{3}{3}\right) = 3 : \frac{24}{4}$$

		5				5			5	

$\Rightarrow$ A : B : C = 2 : 3 :	$\frac{2}{4}$		= 10 : 15 : 24
	5		

$\Rightarrow$ B =	(	98 x	$\frac{15}{49}$	)		= 30.

5. Answer: Option **D**
Explanation:

Given ratio = $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ = 6 : 8 : 9.

$\therefore$ 1st part = Rs.	(	782 x	$\frac{6}{23}$	)		= Rs. 204

HOTS-HIGH ORDER THINKING SKILLS

1. Answer: Option **B**
Explanation:

Let the total profit be Rs. 100.

After paying to charity, A's share = Rs.	(	95 x	$\frac{3}{5}$	)	= Rs. 57.

If A's share is Rs. 57, total profit = Rs. 100.

If A's share Rs. 855, total profit =	(	$\frac{100}{57}$	x 855	)	= 1500.

2. Answer: Option **B**
Explanation:

For managing, A received = 5% of Rs. 7400 = Rs. 370.

Balance = Rs. (7400 - 370) = Rs. 7030.

Ratio of their investments = (6500 x 6) : (8400 x 5) : (10000 x 3)

= 39000 : 42000 : 30000

= 13 : 14 : 10

$\therefore$ B's share = Rs. $\left(7030 \times \frac{14}{37}\right)$ = Rs. 2660.

3. Answer: Option **D**

Explanation:

Ratio of initial investments = $\left(\frac{7}{2} : \frac{4}{3} : \frac{6}{5}\right)$ = 105 : 40 : 36.

Let the initial investments be 105*x*, 40*x* and 36*x*.

$\therefore$ A : B : C = $\left(105x \times 4 + \frac{150}{100} \times 105x \times 8\right)$: (40*x* x 12) : (36*x* x 12)

= 1680*x* : 480*x* : 432*x* = 35 : 10 : 9.

Hence, B's share = Rs. $\left(21600 \times \frac{10}{54}\right)$ = Rs. 4000.

4. Answer: Option **D**

Explanation:

Let C = *x*.

Then, B = *x* + 5000 and A = *x* + 5000 + 4000 = *x* + 9000.

So, *x* + *x* + 5000 + *x* + 9000 = 50000

$\Rightarrow$ 3*x* = 36000

$\Rightarrow$ *x* = 12000

A : B : C = 21000 : 17000 : 12000 = 21 : 17 : 12.

$\therefore$ A's share = Rs. $\left(35000 \times \frac{21}{50}\right)$ = Rs. 14,700.

5. Answer: Option **B**

Explanation:

Let their investments be Rs. *x* for 14 months, Rs. *y* for 8 months and Rs. *z* for 7 months respectively.

Then, 14*x* : 8*y* : 7*z* = 5 : 7 : 8.

Now, $\frac{14x}{8y} = \frac{5}{7} \Leftrightarrow 98x = 40y \Leftrightarrow y = \frac{49}{20}x$

And, $\frac{14x}{5} \Leftrightarrow 112x = 35z \Leftrightarrow z = \frac{112}{2}x = \frac{16}{.}x$

	$7z$		8		35		5	

$\therefore\ x : y : z = x : \frac{49}{20}x : \frac{16}{5}x = 20 : 49 : 64.$